

Points of Concurrency in Triangles Notes

**CONCEPT SUMMARY**

Segments, Lines, Rays, and Points in Triangles				
	Example	Point of Concurrency	Property	Example
perpendicular bisector bisects each side at $90^\circ$		<u>circumcenter</u> point of concurrency with perpendicular bisectors	The circumcenter $P$ of a triangle is equidistant from the vertices of the triangle. $\overline{PB} = \overline{PA} = \overline{PC}$	
angle bisector bisects each vertex or angle		<u>incenter</u> point of concurrency with angle bisectors	The incenter $I$ of a triangle is equidistant from the sides of the triangle. $\overline{IX} = \overline{IY} = \overline{IZ}$	
median connect vertex to midpoint of opposite side		<u>centroid</u> $AR = \frac{2}{3}(AY)$ $CR = \frac{2}{3}(CX)$ $BR = \frac{2}{3}(BQ)$	The centroid $R$ of a triangle is two thirds of the distance from each vertex to the midpoint of the opposite side.	
altitude connect vertex to opposite side at $90^\circ$		<u>orthocenter</u>	The lines containing the altitudes of a triangle are concurrent at the orthocenter $O$ .	

Medians of Triangles

$\overline{AH}$ ,  $\overline{BJ}$ , and  $\overline{CG}$  are medians of a triangle. They each join a vertex and the midpoint of the opposite side.

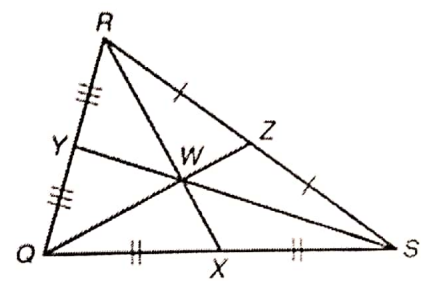
The point of intersection of the medians is called the **centroid** of  $\triangle ABC$ .

Theorem	Example
<p><b>Centroid Theorem</b> The centroid of a triangle is located <math>\frac{2}{3}</math> of the distance from each vertex to the midpoint of the opposite side.</p>	<div style="text-align: center;"> </div> <p>Given: <math>\overline{AH}</math>, <math>\overline{CG}</math>, and <math>\overline{BJ}</math> are medians of <math>\triangle ABC</math>.</p> <p>Conclusion: <math>AN = \frac{2}{3}AH</math>, <math>CN = \frac{2}{3}CG</math>, <math>BN = \frac{2}{3}BJ</math></p>

### Points of Concurrency in Triangles Notes

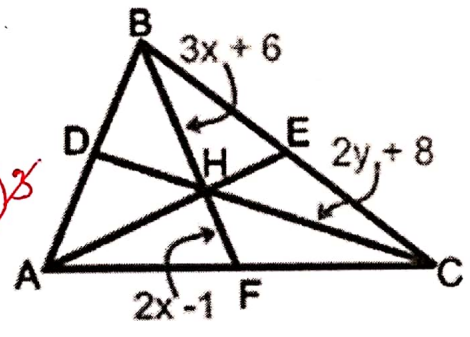
Example: Medians of a Triangle

In  $\triangle QRS$ ,  $RX = 48$  and  $QW = 30$ . Find each length.



1.  $RW =$   
 $x = \frac{2}{3}(48) = 2(16) = 32$
2.  $WX = 48 - 32 = 16$
3.  $QZ$   
 $\frac{3}{2}(30) = (\frac{2}{3}(x)) \frac{3}{2}$
4.  $WZ = 45 - 30 = 15$

$DC = 5y - 16$ . Find  $x$  and  $y$ .



$$CH = \frac{2}{3}(CD)$$

$$3(2y + 8) = \left(\frac{2}{3}\right)(5y - 16) \cdot \frac{3}{2}$$

$$6y + 24 = 10y - 32$$

$$\begin{array}{r} 6y + 24 = 10y - 32 \\ -6y \quad -6y \\ \hline 24 = 4y - 32 \\ +32 \quad +32 \\ \hline 56 = 4y \\ \frac{56}{4} = \frac{4y}{4} \quad \boxed{y = 14} \end{array}$$

$$BH = \frac{2}{3}(BF)$$

$$3(3x + 6) = \left(\frac{2}{3}\right)(5x + 5) \cdot \frac{3}{2}$$

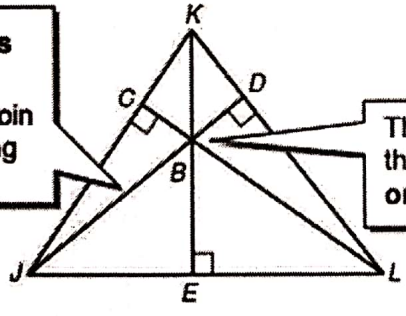
$$\begin{array}{r} 9x + 18 = 10x + 10 \\ -9x \quad -9x \\ \hline 18 = x + 10 \\ -10 \quad -10 \\ \hline x = 8 \end{array}$$

$$BF = 3x + 6 + 2x - 1 = 5x + 5$$

Altitudes of Triangles

$x = 8$

$\overline{JD}$ ,  $\overline{KE}$ , and  $\overline{LC}$  are altitudes of a triangle. They are perpendicular segments that join a vertex and the line containing the side opposite the vertex.



The point of intersection of the altitudes is called the orthocenter of  $\triangle JKL$ .