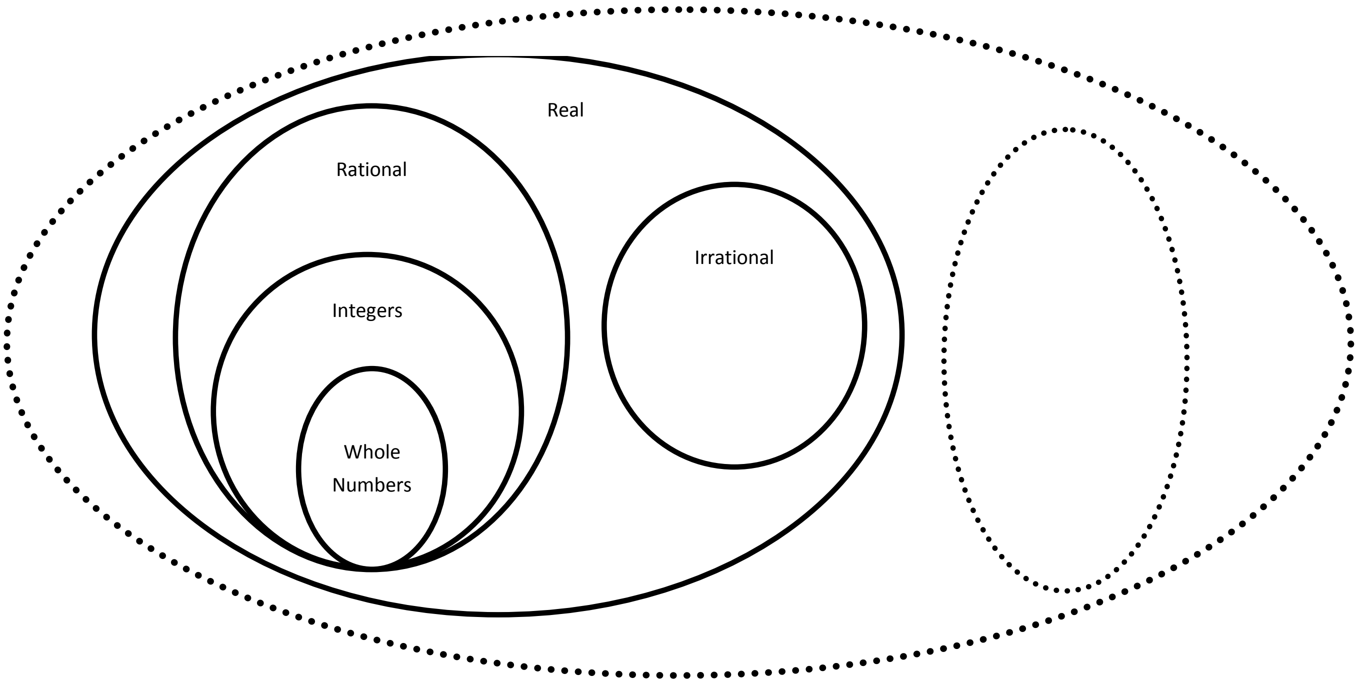
Name: Period: Date:

Math Lab: Investigating Imaginary Numbers

Question

How can you define square roots of negative numbers?

Recall the Real Number System



Investigate powers of *i*

The square roots of 1 are 1 and –1, but are there any "square roots" for –1? Mathematicians have defined a number i so that $i = \sqrt{-1}\ and\ i^2 = -1$.

Complete the table for powers of i.

Power	**Expansion**	**Result**
i^1	i	i
i^2	$(i) \cdot i =$	
i^3	$(i \cdot i) \cdot i = i^2 \cdot i =$	
i^4	$(i \cdot i \cdot i) \cdot i = i^3 \cdot i =$	
i^5	$(i \cdot i \cdot i \cdot i) \cdot i = i^4 \cdot i =$	
i^6		
i^7		
i^8		

- Identify and describe any patterns you see in the results. Use the pattern to predict $i^9, i^{10}, i^{11}, i^{12}$.

➢ Describe how you can identify each of the following where n is a positive integer.

Values of n for which $i^n = 1$.

Values of n for which $i^n = i$.

Values of n for which $i^n = -1$.

Values of n for which $i^n = -i$.

➢ Find the simplified form of each power of i. Explain your reasoning.

$i^{16} =$	$i^{101} =$	$i^{999} =$

Investigate square roots of negative numbers

For any nonnegative real number r, $(\sqrt{r})^2 = r$. Assume this property holds true for negative real numbers.

$(\sqrt{-2})^2 =$	$(i\sqrt{2})^2 = i^2(\sqrt{2})^2 =$
$(\sqrt{-3})^2 =$	$(i\sqrt{3})^2 = i^2(\sqrt{3})^2 =$
$(\sqrt{-4})^2 =$	$(i\sqrt{4})^2 =$
$(\sqrt{-5})^2 =$	$(i\sqrt{5})^2 =$

➢ What do you notice about the simplified forms of the expressions in each row?

➢ Write an expression involving i that is equal to $\sqrt{-7}$.

➢ If x is a square root of a number, then $-x$ is also a square root of the number, because $(-x)^2 = x^2$. Apply this property to find the square roots of $\sqrt{-36}$.