

## Notes: Factoring Polynomials Completely

Polynomial equations, that are non-linear (degree of largest exponent is greater than 1) need to be in factored form in order to apply to zero product property and solve for the variable.

Often these polynomial equations may require more than one factoring method for the polynomial to be completely factored.

Order:

1. Factor out any GCF (Greatest Common Factor)
2. If there are four terms, try factor by grouping
3. If there are two terms, try factor by difference of squares
4. If there are three terms, try factor by diamond and grouping **OR** check if it is a perfect square trinomial.

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^2 + 2ab + b^2 = (a + b)(a + b)$$

$$a^2 - 2ab + b^2 = (a - b)(a - b)$$

Examples:

Completely factor the polynomial expressions.

1.  $24x^3 - 12x^2 - 20x + 10$   
 $2(12x^3 - 6x^2 - 10x + 5)$   
 $2[6x^2(2x - 1) - 5(2x - 1)]$   
 $2(2x-1)(6x^2-5)$   
Factor out GCF  
Factor by grouping within the parenthesis  
**Completely factored form**
2.  $5x^4 - 80$   
 $5(x^4 - 16)$   
 $5(x^2 - 4)(x^2 + 4)$   
 $5(x-2)(x+2)(x^2 + 4)$   
Factor out GCF  
Factor difference of squares  $x^4 - 16$   
Factor difference of squares  $x^2 - 4$   
**Completely factored form**
3.  $10x^2 - 31x + 15$   
 $10x^2 - 25x - 6x + 15$   
 $5x(2x - 5) - 3(2x - 5)$   
 $(2x-5)(5x-3)$   
Factor trinomial using diamond to split middle term  
Factor by grouping  
**Completely factored form**
4.  $27x^2 - 90x + 75$   
 $3(9x^2 - 30x + 25)$   
 $3(3x-5)(3x-5)$   
Factor out GCF  
Factor perfect square trinomial ( $a = 3x, b = 5, 2ab = 2(3x)(5) = 30x$  ✓ checked)  
**Completely factored form**

## Practice Problems

Completely factor the following polynomial expressions. Answers are at bottom of page in no particular order.

|                      |                             |
|----------------------|-----------------------------|
| 1. $-8x^2 - 10x - 3$ | 2. $200 - 32x^{10}$         |
| 3. $2x^2 + 26x + 80$ | 4. $12x^3 - 2x^2 + 3 - 18x$ |
| 5. $3x^2 + 5x - 9$   | 6. $4x^2 - 20x + 25$        |
| 7. $8x^2 - 16x + 6$  | 8. $3x^2 - 3xy + 24x - 24y$ |

Answers in no particular order:

$(2x - 5)(2x - 5)$

$3(x - y)(x + 8)$

$2(x + 5)(x + 8)$

*unfactorable*

$2(2x - 3)(2x - 1)$

$(4x + 3)(-2x - 1)$

$8(5 + 2x^5)(5 - 2x^5)$

$(6x - 1)(2x^2 - 3)$